$$\left[a_1\frac{d^2}{dy^2}+b_1\right]e-\left[c_1\frac{d}{dy}+d_1\right]h=0 \quad (12a)$$

$$\left[c_2\frac{d}{dy}-d_2\right]e+\left[a_2\frac{d^2}{dy^2}+b_2\right]h=0 \quad (12b)$$

where the coefficients are defined by comparison to (3) and (4).

The separated equations allow rigorous solution for any set of boundary conditions, such as the set arising in a partially filled guide. For the completely filled waveguide, by (5) and (6), u and v' must vanish on $x=(0, x_0)$ in *both* types of waveguides. Thus the solutions for (11a) and (11b) are

$$u=\sin\alpha_m x, \qquad v=\cos\alpha_m x$$

with $\alpha_m=c=c'=m\pi/x_0$. The solutions of (12a) and (12b) are found to be combinations of trigonometric functions.

The conditional equation is obtained in the usual manner. After introducing the boundary conditions, one obtains two algebraic equations in two unknowns whose solution exists only if the determinant vanishes. In gyroelectric waveguide, the condition of vanishing e and h' on $y=(0, y_0)$ leads to the set of equations whose determinant is

$$\begin{aligned}P(\kappa, H_0) &= \frac{1}{(s_1^2-s_2^2)^2}\Big\{2\alpha_m^2\epsilon_2^2[\cos s_1y_0\cos s_2y_0-1] \\ &-\frac{1}{s_1s_2}\left[\frac{\epsilon_1}{k^2}(k_1^2-\alpha_m^2)(s_1^2-s_2^2)^2\right. \\ &\left.-\alpha_m^2\epsilon_2^2(s_1^2+s_2^2)\right]\sin s_1y_0\sin s_2y_0\Big\} \quad (13)\end{aligned}$$

where

$$\begin{aligned}s_1^2+s_2^2 &= -2\kappa^2 - \frac{1}{\epsilon_1}[(\alpha_m^2-k^2\epsilon_1)(\epsilon_1+1)+k^2\epsilon_2^2] \\ s_1^2-s_2^2 &= \frac{1}{\epsilon_1}\{[(\alpha_m^2-k^2\epsilon_1)(\epsilon_1-1)+k^2\epsilon_2^2]^2 \\ &+4\alpha_m^2k^2\epsilon_2^2\}^{1/2}.\end{aligned}$$

It is evident that evaluation of the roots of $P(\kappa, H_0)$ is a difficult problem. An attempt at expressing the roots κ as a Taylor series in powers of H_0 was not successful. In this series, the derivatives of κ were evaluated by implicit differentiation of $P(\kappa, H_0)$. Since the theory of implicit functions is valid for single-valued functions only, and since κ is a double-valued function of H_0, this method is not applicable. It is interesting that the second-order Taylor approximation gives the arithmetic mean of the two branches of κ calculated from degenerate perturbation theory. We also note that the limiting form of P for the $TE_{0,n}$ modes is obtained readily by setting $\alpha_m=0$, the value of α corresponding to no variation in x. The roots are then the correct ones given in (7). On the other hand, to find the roots for the wave to which the $TE_{m,0}$ mode is distorted, one needs to trace the variation of P as H_0 is changed from zero. This is a complicated process since every term in P is a function of H_0.

Conclusion

We have seen that, although gyromagnetic and gyroelectric waveguides are duals, their behaviors are different. The differences are particularly significant in the $TE_{0,n}$ modes. Gyroelectric phenomena in rectangular waveguide, unlike gyromagnetic phenomena, can be investigated only through the general solution which is too unwieldy for practical applications. Thus, approximation methods [5] are very desirable. We have also shown that the general solution can be obtained rigorously by the method of separation of variables.

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A High Directivity, Broadband Coaxial Coupler

Directional couplers having the proper impedance and directivity characteristics are useful devices for reflectometry measurements and for the intercomparison of RF power meters. In the latter application, stable, sensitive power detectors are connected to the appropriate coupler sidearms to allow monitoring of both the incident and reflected powers in the primary line.

While it is possible to achieve high directivity with ordinary couplers at a given frequency by use of auxiliary tuners, the tuning procedures are somewhat involved and time-consuming. It is thus more convenient to use couplers which are of inherent high directivity. In addition, tuning for high directivity may aggravate the impedance mismatch conditions at one or more of the coupler output ports.

It is the purpose of this correspondence to describe the design of a high (50-dB) directivity broadband (0.7 to 1.5 and 3 to 4 GHz) coaxial coupler having low VSWR (<1.01) in both the primary and secondary lines. Achievement of these characteristics was made possible to a large degree by the recent availability of precision coaxial connectors which are used on all ports. The basic design is more or less conventional in that two transmission lines are inductively and capacitively coupled by means of a common slot in their outer conductors. Inductive coupling is a function of the width of the slot, while capacitive coupling is a function of the spacing between center conductors. The coupling ratio K, as derived by Monteath,[1] is given by

$$K=\frac{1}{2}\left(k-\frac{1}{k}\right)\sin\beta l \qquad (1)$$

where βl is the length of the coupling slot in electrical wavelengths and k is a constant determined by the slot width and the spacing between center conductors. Thus, the coupling is a maximum at frequencies (f_c, $3f_c$, etc.) where l is an odd multiple of a quarter-wavelength, $\lambda/4$. The coupling is near zero at frequencies where l is an even multiple of $\lambda/4$. The bandwidth of the coupler, centered on each of the frequencies f_c, $3f_c$, etc., is roughly equal to f_c (i.e., 1150 MHz for the coupler described here).

High directivity was obtained primarily by designing the coupler so that the proper balance was obtained in the mutual capacitance and inductance between the coupled lines. Methods for calculating the proper spacing dimensions for a given coupling ratio are given by Monteath.[1] Of almost equal importance for broadband use is the elimination of impedance discontinuities within the coupler, at connectors, and in the load resistor which terminates the secondary line. Discontinuities cause reflections which, even though small, can significantly reduce the directivity. For example, in order to achieve 50-dB broadband directivity it is necessary that the reflection coefficient of internal discontinuities be reduced to the order of 0.002 or less. Usually, the largest discontinuity in coaxial couplers with coupling closer than about 30 dB occurs at the coupling slot. In this region the characteristic impedance Z_0 of both the primary and secondary lines tends to be lower than in the uncoupled region. In the 20-dB, 50-ohm coupler described here, the Z_0 in the coupled region as measured with a time domain reflectometer (TDR) was found to be 46 ohms. A wave propagating from the input would see an abrupt change in line impedance from 50 ohms to 46 ohms at the beginning of the coupling slot. The impedance remains 46 ohms along the length of the slot and then abruptly changes to 50 ohms again at the end. In order to bring the impedance in the coupled region back to 50 ohms, it was necessary to increase the effective outer to inner conductor diameter ratio. This was done by undercutting the outer conductors in the coupled region with a milling tool having a diameter (for convenience) that was the same as the original line diameter. Most of the undercutting was done in the lower and upper halves, respectively, of the primary and secondary outer conductors. Thus, in cross section, the shape of the outer conductors tended to become oblong in shape as shown in Fig. 1.

The coupler was made of brass with an outer conductor diameter of 0.5625 inch and

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[1] G. D. Monteath, "Coupler transmission lines as symmetrical directional couplers," *Proc. IEE (London)*, pt. B, vol. 102, pp. 383–392, May 1955.

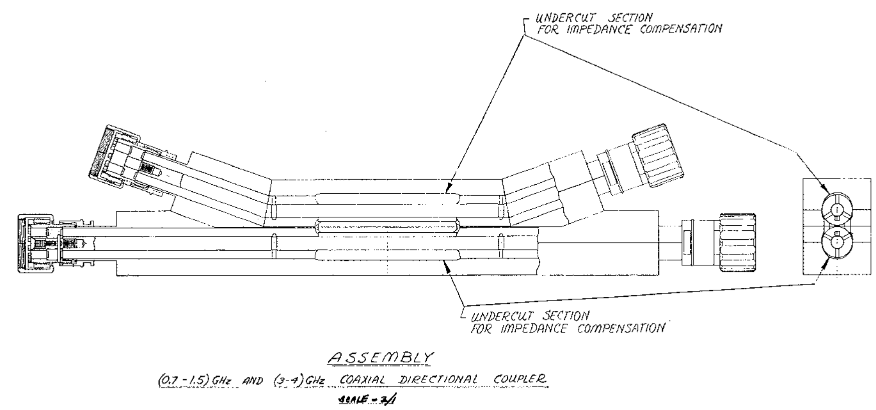

Fig. 1. Section view of coupler.

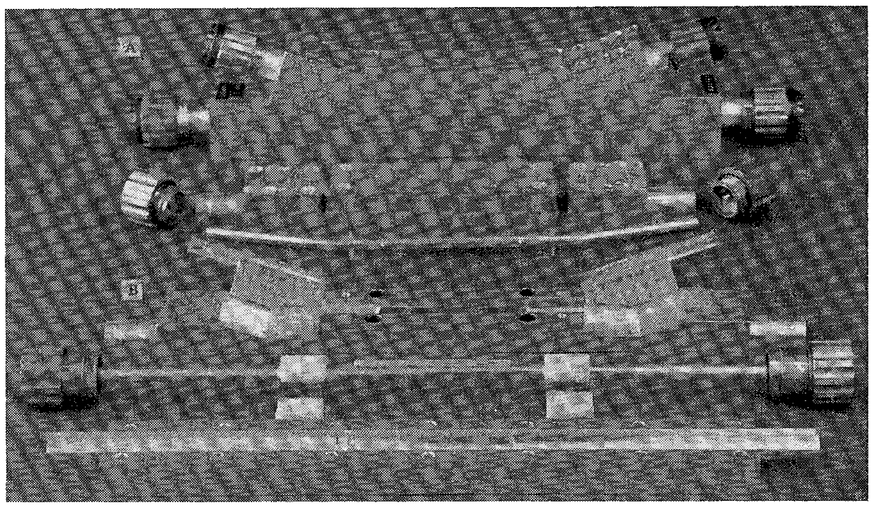

Fig. 2. Photograph of coupler. (A) Assembled. (B) Disassembled.

an inner conductor diameter of 0.2442 inch. Outputs for the secondary line were brought out at an angle of 20° to the primary and GR type 900 BT precision connectors were used on all four ports. The Z_0 of both lines was made 50 ± 0.1 ohms throughout their length by repeated checking with the TDR and making the necessary adjustments in line dimensions to eliminate discontinuities. The entire assembly was silver-flashed after all adjustments were completed.

The length of the coupling slot was cut for an f_c of 1150 MHz and the measured (± 1 dB) directivity is 50 dB in the ranges 750 to 1700 MHz and 3000 to 4000 MHz. The equivalent residual reflection coefficient, $|\Gamma_r|$, is thus 0.0031 in these frequency bands. The quantity $|\Gamma_r|$ is defined as the ratio of output voltages, $|b_3|/|b_4|$, at the secondary ports which sample, respectively, the reflected and incident waves with the primary output port terminated in a matched load. Small tuning slugs located in the secondary outer conductor near the 20° bends allow the directivity to be increased to 60 dB within narrow bands (50 to 100 MHz) in the above frequency ranges. Figure 2 is a photograph of the coupler.

This coupler was designed for use primarily as a transfer device for the intercomparison of absorption type RF power meters. Such intercomparisons are usually carried out by alternate connection of the meter under test and the standard meter to a stable power generator. Due to impedance mismatch, the powers delivered to the meter and the standard may not be the same. When the complex reflection coefficients, Γ_m, Γ_s, and Γ_g of the meter, standard, and generator are known, it is possible to calculate the ratio, $K = P_m/P_s$, of the powers delivered to the meter and the standard by use of the equation[2]

$$K = \frac{P_m}{P_s} = \left|\frac{1 - \Gamma_g\Gamma_s}{1 - \Gamma_g\Gamma_m}\right|^2 \frac{1 - |\Gamma_m|^2}{1 - |\Gamma_s|^2}. \quad (2)$$

If, as is often the case, only VSWR information is available, then the value of K cannot be calculated exactly but rather its upper and lower limits may be determined from

$$\frac{r_m}{r_s}\left(\frac{r_g r_s + 1}{r_g + r_m}\right)^2 \geqq K \geqq \frac{r_m}{r_s}\left(\frac{r_g + r_s}{r_g r_m + 1}\right)^2, \quad (3)$$

where r_m, r_s, and r_g are the VSWR's of the meter, standard, and generator, respectively. The uncertainty in K is an error in the intercomparison process, and hence, it is highly desirable to reduce it to as low a value as possible. Note that the limits of K close much more rapidly when $r_g \to 1$ than is the case when either of the other variables, r_s or r_m, approach 1. In the limit, if r_g can be made equal to unity, then (3) reduces to

$$K = \frac{r_m}{r_s}\left(\frac{r_s + 1}{r_m + 1}\right)^2 \quad (4)$$

and the ambiguity is resolved.

It has been shown by Engen[3] that the combination of a directional coupler with a power monitor connected to the proper sidearm and a suitable feedback system to the RF source to maintain a constant incident power can become the equivalent of a matched generator. In terms of the scattering coefficients, S_{ij}, for a 4-arm coupler, the equivalent generator reflection coefficient is given by

$$\Gamma_g = \left(S_{22} - \frac{S_{12}S_{13}}{S_{13}}\right). \quad (5)$$

In this equation, S_{22} is the reflection coefficient looking back into the primary output port with all other arms terminated in matched loads. The term $|S_{23}|/|S_{13}|$ is the directivity, while $|S_{12}|$ is the coupling between primary input and output ports and is less than unity.

The degree of equivalent match thus depends on how closely (5) approaches zero. In the coupler described here, $|S_{22}|$ is 0.005 (VSWR = 1.01), while $|S_{23}|/|S_{13}|$ for 50-dB directivity is 0.0031. Assuming worst phase conditions, the equivalent generator reflection coefficient is then 0.0081 corresponding to a VSWR of 1.016. This value was verified (with the limits of experimental error at 3 GHz) by direct measurement. By comparison, typical high quality commercially available couplers have 30-dB directivity and primary line VSWR of 1.05 which gives an equivalent generator VSWR of the order of 1.12.

In using the new coupler to intercompare power meters with the National Bureau of Standards calorimetric reference standard power meter (VSWR < 1.01 to 3 GHz), the maximum transfer uncertainty due to mismatch is 0.1 percent for meters having VSWR's up to 1.10. The reference standard is used primarily for the calibration of working standards for the NBS Electronic Calibration Center. These have VSWR's of less than 1.05. The Electronic Calibration Center will also use the couplers in the calibration of customer interlaboratory standards.

Further information, including machine drawings, is available from National Bureau of Standards, Radio Standards Engineering Division, Boulder, Colo.

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